Wireless-Aware Nonlinear Model Predictive Control for Mobile Robot Navigation over IEEE 802.11ax Networks

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Abstract
Closing the control loop of an autonomous mobile robot over commercial IEEE 802.11ax (Wi-Fi 6) infrastructure exposes NMPC to stochastic, temporally correlated observation delays. We make three contributions. *First*, we characterize per-packet latency for a warehouse Wi-Fi 6 deployment using Sionna-based PHY simulation and identify a **TWT resonance effect**: service periods that are integer multiples of the control period dt achieve P99_dev ≈ 2 ms, whereas non-harmonic SP = 190 ms yields P99_dev ≈ 91 ms and Geometric($p = 0.13$) burst delays with mean L■ = 6.4 steps. *Second*, we prove that standard NMPC is naturally robust to i.i.d. delays with P99_abs ≤ N·dt (Proposition 2), but that burst delays of length $L > N$ accumulate position error beyond the planning margin (Corollary 2). *Third*, we propose **ETDA-NMPC** and derive Corollary 3: it is tube-feasible iff Γ_max = max P99_dev(j·dt)/dt < 1/(2 L_f L_ctrl dt). Closed-loop IPOPT simulation ($n = 5$, $T = 200$ steps) shows: (i) i.i.d. worst-case delay causes only 0.3% violations (horizon recovery); (ii) burst delay causes 3× more violations; (iii) fixed-τ shift alone worsens minimum clearance by 36 mm; (iv) ETDA-NMPC achieves 0.0% violations and 10× better minimum clearance.

Keywords: *NMPC, IEEE 802.11ax, Wi-Fi 6, networked control, mobile robotics, TWT, stochastic delay, ETDA-NMPC, tube MPC, burst delay*

I. INTRODUCTION

Autonomous mobile robots in industrial environments rely on commercial Wi-Fi to close the control loop. IEEE 802.11ax (Wi-Fi 6) introduces OFDMA and Target Wake Time (TWT) scheduling, yet the NCS literature treats delays as i.i.d. Gaussian or worst-case bounded [1, 3]. These abstractions miss the structured *burst* behavior of 802.11ax MAC-layer TWT scheduling: when a control packet falls between TWT wake windows, it waits until the next window, creating Geometric(p=0.13) burst lengths with temporal correlation $\rho = 0.81$ at SP = 190 ms.

NMPC's N-step prediction horizon provides natural robustness to *isolated* large delays: a single 90 ms delay introduces 1 stale step (at dt = 100 ms), absorbed within 3 steps. However, a burst of mean length L■ = 6.4 steps saturates the prediction horizon, causing the controller to plan from a position 0.77 m behind the actual robot.

A. Contributions

- **TWT resonance characterization:** 16-SP Sionna sweep; Geometric(p=0.13) burst calibration; $\rho = 0.81$ at non-harmonic SP = 190 ms.
- **Formal robustness bounds (Props. 1–2, Cor. 2):** Disturbance amplitude from empirical CDFs; i.i.d. horizon-recovery proof; burst saturation bound.
- **ETDA-NMPC feasibility certificate (Cor. 3):** Closed-form condition Γ_max < 1/(2 L_f L_ctrl dt) establishing SP = n·dt as feasibility precondition.
- **Closed-loop safety validation:** 10× better minimum clearance; characterization of fixed-τ failure mode.

II. RELATED WORK

Heemels et al. [1] established NCS stability theory under packet dropouts; the delay model is abstract with no wireless PHY characterization. Park et al. [3] proposed cross-layer co-design for 802.11n with simplified i.i.d. models. Zhang et al. [4] analyzed stability under Markov-modulated delays; our contribution replaces the Markov model with Geometric burst lengths calibrated to Sionna measurements.

Mayne et al. [6] formulated standard Tube-MPC; Rakovic et al. [7] derived minimal robust positively invariant sets. Our work derives the disturbance set W from empirical 802.11ax CDFs. No prior work connects P99_dev from a specific TWT configuration to a closed-loop NMPC safety certificate.

III. SYSTEM MODEL

A. Robot Dynamics

Unicycle state x = [px, py, θ]■, input u = [v, ω]■, ■ = [v cosθ, v sinθ, ω]■ (1), discretized with RK4 at dt = 0.1 s. Constraints: v ∈ [0, 1.2] m/s, ω ∈ [–1.5, 1.5] rad/s.

Lemma 1 (Lipschitz). *The RK4 flow map satisfies ■F(x,u) – F(x′,u)■ ≤ L_f ■x – x′■ with L_f ≤ 1.01 at dt = 0.1 s.*

B. TWT Burst Process

Let d_k ≥ 0 denote one-way sensor-to-controller latency; the controller receives x■_k = x_{k–m_k} where m_k = ■d_k/dt■. Under non-harmonic TWT:

Definition 1 (Burst Process). *Burst onset: Bernoulli with rate λ = 1/T_lcm, T_lcm = lcm(SP, dt)/dt steps. Burst length: L ~ Geometric(p), calibrated from Sionna. During burst: d_k ~ Uniform(50 ms, P99_abs); outside: d_k ~ N(d■_res, 0.5 ms). For SP=190ms: T_lcm=19, p=0.13, ρ=0.81, L■=7.7, P(L ≥ N) = 0.23.*

Remark 1. *The first-order Markov model d_k = ρ d_{k–1} + (1–ρ)d■_k is inappropriate for non-harmonic TWT. The phase-walk is deterministically periodic with period T_lcm, not a continuous-time Markov process.*

Lemma 2 (Drift Bound). *■(p_k – p■_k)■■ ≤ v_max · d_k.*
Proof: ■p_k – p_{k–m}■ ≤ Σ dt|v_i| ≤ m·dt·v_max = d_k·v_max. ■

IV. IEEE 802.11AX LATENCY CHARACTERIZATION

A. Simulation Setup

Sionna 0.17 (TF 2.13) + ns-3 v3.40 (WIFI_STANDARD_80211ax). PHY: TGax Model D (warehouse NLOS, ≥50m), 5.2 GHz, 20 MHz, MCS 7 (64-QAM, R=5/6), SNR = 20 dB. HW impairments: AGC 3 μs, phase noise 1° rms, CFO 10 ppm. EDCA (AC_BE): CWmin=15, CWmax=1023, AIFSN=3, TXOP=2.528 ms. Traffic: CBR, Δt=100 ms, 200 B payload; GI = 0.8 μs HE Short. Retransmission: MaxSlrc=7, app-level cap=3. Sweep: N_pkt=2000, 16 SP ∈ [50,200] ms, STA ∈ {8, 32}.

B. TWT Resonance Effect

Proposition 1 (Resonance Condition). *Let r = SP/dt. If r ∈ ■■, queuing delay d_queue ≤ dt. If r ∉ ■■, the phase-walk period is T_lcm steps and E[d_queue] = SP(1 – 1/■r■)/2. For r = 1.9: E[d_queue] = 42.75 ms.*

TABLE I — 802.11ax Latency Statistics (8 STAs, selected SP)

SP (ms)	Mean (ms)	P99 (ms)	P99_dev (ms)	ρ	p■	Regime
50	2.15	3.04	1.94	0.04	—	r=0.5, resonant
100	2.14	3.03	1.92	0.05	—	r=1, resonant
150	2.42	51.38	49.71	0.67	0.29	r=1.5, mild
190	92.22	182.69	90.73	0.81	0.13	r=1.9, worst
200	52.15	103.00	51.83	0.52	0.21	r=2, resonant

C. Non-Stationarity of Γ_max

Definition 2 (Feasibility Ratio). *$\Gamma\_max \equiv max\_\{j \in \{0,\dots,T\_lcm-1\}\}\ P99\_dev(j \cdot dt)/dt$. For SP=100ms: T_lcm=1, Γ_max=0.019 (stationary). For SP=190ms: T_lcm=19, Γ_max oscillates 0.02 → 0.91 over 19-step cycle.*

Remark 2 (Commissioning). *Measure P99_dev over ≥ T_lcm consecutive control periods before certifying compliance. For SP=190ms: T_lcm=19 → 1.9 s minimum observation. For SP=100ms: 1 step = immediate.*

V. WIRELESS-AWARE NMPC DESIGN

A. NMPC Formulation

min_{U_k} Σ_{i=0}^{N–1} ■(x,u) + V_f(x_{k+N}) s.t. x_{i+1}=F(x_i,u_i), x_0 = x■_k, u ∈ U, x ∈ X_δ. N=10, Q=diag(10,10,1), R=diag(1,0.5). Obstacle avoidance: soft penalty w_obs max(0, r_j+δ_plan–■p–c_j■)² per obstacle, w_obs=2000. Solver: CasADi+IPOPT, warm-start, ~30 ms/step.

B. Disturbance Bound

Proposition 2 (Disturbance Bound). *Let δ_ unicycle with L_f ≤ 1.01 and controller Lips ■Δ_k■■ ≤ L_f · L_ctrl · |δ_k|. Setting |δ_k| L_ctrl · P99_dev.*

Remark 3 (Conservatism). *Analytical L_ctrl is 3.4× larger than the empirical estimate L from IPOPT logs. A tighter empirical bound 1.10 m to ≈ 0.32 m.*

C. i.i.d. Horizon Recovery

Proposition 3 (i.i.d. Recovery). *If d_k ≤ D_max ≤ N·dt a.s. and delays are i.i.d., then for any single large delay d_k = D_max: ■x_{k+m} – x*_{k+m}■ ≤ c λ^m D_max, m ≥ 0, for c>0, λ∈(0,1). Recovery occurs within ■D_max/dt■+1 steps.*

Corollary 2 (Burst Saturation). *If a burst of length L > N occurs, accumulated position error satisfies ■p_{k+L} – p■_{k+L}■ ≥ (L–N)·v_min·dt. For SP=190ms: P(L ≥ N) = 0.87^10 = 0.23; 23% of bursts saturate the horizon.*

D. ETDA-NMPC Feasibility Condition

Corollary 3 (Tube Feasibility). *ETDA-NMPC with tightening ε is tube-feasible iff:*

$$\Gamma\_max < 1 / (2 \cdot L\_f \cdot L\_ctrl \cdot dt) \approx 0.413 \qquad (4)$$

SP=100ms: Γ_max=0.019 < 0.413 ✓ (feasible). SP=190ms: Γ_max=0.907 > 0.413 ✗ (infeasible by 2.2×). Thus SP = n·dt is a feasibility precondition for ETDA-NMPC.

E. ETDA-NMPC Algorithm

Algorithm 1: ETDA-NMPC (per control step k)

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Input: x■_k, P99_abs, P99_dev
1: Assert Γ_max < threshold; else fall back to
standard NMPC
2: τ ← ■P99_abs/dt■; x*_ref ← x*_{k+τ} ■
look-ahead
3: ε ← L_f · L_ctrl · P99_dev
4: v_max^ETDA ← v_max(1 – ε/(v_max · N · dt))
5: δ_plan ← δ_phys + ε/2
6: Solve NMPC (eqn 2) with (x*_ref, v_max^ETDA,
δ_plan)
7: Apply u_k = u*_{k|k}
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Why fixed-τ alone fails (S5): A look-ahead shift targets a waypoint that, on curved lemniscate segments, lies closer to an obstacle than the current reference. Without constraint tightening (step 5), the optimizer accepts reduced clearance. S5 shows 0.5% violations but minimum clearance = –7 mm (surface penetration).

VI. EVALUATION

A. Setup

CasADi+IPOPT NMPC, N=10, warm-start, T=200 steps (20 s). Reference: figure-8 lemniscate (scale=1.8m, ω_r=0.314 rad/s). Three circular obstacles at (±0.9, 0.0) and (0.0, +0.6), r=0.3m, placed near figure-8 crossing/inflection points. Delay injection: x■_k = x_{k–m_k} from rolling state buffer (sensor-to-controller latency model). **Primary safety metric:** mean minimum obstacle clearance (more informative than violation rate under soft constraints).

B. Results

TABLE II — Closed-Loop Performance (n=5 rollouts, T=200 steps, v_max=1.2 m/s, δ_phys=0.06 m)

Cond	Description	RMSE (m)	±std	Viol. (%)	±std	Min Clr (m)	Max Con
S1	No delay (baseline)	0.394	0.000	1.5	0.0	+0.011	2.0
S2	TWT-resonant i.i.d.	0.394	0.000	1.5	0.0	+0.011	2.0
S3	TWT-worst i.i.d. (no burst)	0.389	0.006	0.3	0.4	+0.071	0.6
S4	TWT-worst burst (p=0.13)	0.392	0.010	0.9	0.5	+0.030	1.6
S5	Fixed-τ NMPC (shift only)	0.377	0.008	0.5	0.5	–0.007	0.8
S6	ETDA-NMPC, burst (Γ■thr.)	0.399	0.008	0.0	0.0	+0.109	0.0
S7	ETDA-NMPC, resonant (Γ■thr.)	0.401	0.000	0.0	0.0	+0.102	0.0

S1–S5: v_max=1.2 m/s, δ_plan=0.08 m. S6, S7: v_max^ETDA=0.96 m/s, δ_plan=0.155 m, τ=2. Min. clearance < 0 = surface penetration (S5). Green: zero violations. Orange: partial improvement.

C. Analysis

S1 and S2: TWT-resonant delay (Γ_max=0.019, ≤1 step, no burst) is transparent to NMPC: S2 ≡ S1 to machine precision. The 1.5% residual violations in S1 are a soft-constraint artifact (weight ratio w_obs/Q = 200); a hard-constraint formulation would yield 0.0%.

S3 vs. S4 (i.i.d. vs. burst): S3 shows *fewer* violations (0.3%) than the no-delay baseline (1.5%), confirming Proposition 3: i.i.d. delays with P99_abs=183ms → m_k ≤ 2 stale steps ■ N=10; receding-horizon correction smooths trajectories. S4 (geometric burst, p=0.13, L■=7.7) shows 3× more violations than S3 (0.9% vs 0.3%) and a 41 mm drop in minimum clearance, consistent with Corollary 2: P(L ≥ N) = 0.23 of bursts saturate the horizon.

S5 (fixed-τ): Violation rate improves to 0.5% but minimum clearance drops to –7 mm (surface penetration), the worst in the study. On curved lemniscate segments, the τ=2 look-ahead targets a waypoint closer to the obstacle without tightening margins. This confirms that reference shift alone is insufficient and potentially counterproductive.

S6 and S7 (ETDA-NMPC): Both achieve 0.0% violations and minimum clearance >100 mm — 10× improvement over the S1 baseline (+11 mm). RMSE cost: +2% under TWT resonance (S7 vs S2), +1% under burst (S6 vs S4). The infeasibility distinction of Corollary 3 does not manifest at this operating point because the wider planning margin (0.155 m) provides sufficient clearance in both regimes. It becomes critical at higher v_{max} or in tighter environments where constraint tightening would render the NLP infeasible.

VII. DISCUSSION

A. Co-Design Guidelines

Guideline 1 (TWT configuration): *Set SP = n·dt, n ∈ ■■. This is a* ***feasibility precondition****, not a performance preference. Non-resonant SP causes 3× more burst violations (S4 vs S3) and makes tube-feasibility impossible (Corollary 3). Implementation: Standard Individual TWT negotiation (IEEE 802.11ax-2021 §9.4.2.197); no firmware modification. AP scheduler granularity is the practical constraint (e.g., Cisco CW9166, Aruba AP-635 support ms-level granularity). Certification: Observe P99_dev for ≥ T_lcm = lcm(SP,dt)/dt steps (1.9 s for SP=190 ms) before certifying Γ_max.*

Guideline 2 (Horizon sizing): *If TWT realignment is unavailable, use N ≥ 2 L■ = 15.4 → N ≥ 16 (2.5× IPOPT cost). TWT realignment is computationally free.*

Guideline 2b (OS jitter): *PREEMPT_RT Linux (cyclictest verified): jitter 1–5 ms. Negligible vs TWT-worst but can push resonant P99_dev ≈ 2 ms toward Γ_max threshold by up to 50%.*

Guideline 3 (Safety margin): *Under resonant TWT, ETDA-NMPC costs only +2% RMSE while achieving 10× better minimum clearance. Correct TWT configuration is the more effective intervention than increasing planning margins.*

B. i.i.d. vs. Burst: The Regime Transition

The result that i.i.d. TWT-worst delay (S3) is safer than the no-delay baseline (S1) is counter-intuitive but follows from Proposition 3: NMPC with N=10 horizon absorbs single delays up to N·dt=1 s, smoothing trajectories. Burst delay breaks this because L■=7.7 > N/2: sustained stale observations prevent receding-horizon correction from completing before the next large delay.

Implication: Wi-Fi robot control systems must be validated against burst delay models. The S3/S4 comparison shows i.i.d. worst-case is a 2.4× over-optimistic proxy for burst-realistic worst-case.

C. Limitations

- Soft constraints: 1.5% baseline violation is an artifact. Hard-constraint formulation would yield cleaner delay attribution.
- Conservative L_ctrl: analytical bound (12 s■¹) is 3.4× larger than empirical (3.5 s■¹); tighter estimate would reduce ETDA planning margin by 3×.
- Unicycle model: Lemma 1 requires re-derivation for higher-order dynamics.
- Static STA positions: hardware deployment adds mobility-induced SNR variation.

VIII. CONCLUSION

We presented ETDA-NMPC, a wireless-aware co-design methodology grounded in empirical 802.11ax latency characterization. Four findings: (1) TWT resonance reduces P99_dev by 47× and eliminates geometric burst delays; (2) standard NMPC naturally absorbs i.i.d. delays within N·dt but fails under burst delays with L > N; (3) reference shift alone worsens minimum clearance; combined tightening is required; (4) ETDA-NMPC achieves 10× better minimum clearance at +2% RMSE under TWT resonance. **Central takeaway:** TWT SP configuration is a feasibility decision for delay-aware NMPC, not a tuning parameter.

A. PROOF OF PROPOSITION 2

Setup: x_{k+1}=F(x_k,u_k), u_k=π(x■_k), x■_k=x_{k–m}. Step 1: By Lemma 1, ■F(x,u)–F(x′,u)■ ≤ L_f ■x–x′■. Step 2: Assume π Lipschitz with L_ctrl over the feasible region; then ■u_k–u_k^nom■ ≤ L_ctrl ■x■_k–x_k■. Step 3: ■Δ_k■ ≤ Σ_{i=0}^{m-1} L_f^i ■u_{k-m+i}–u^nom■·dt ≤ L_f · L_ctrl · m · dt = L_f · L_ctrl · d_k. Replacing d_k with |δ_k| gives Eq. (3). ■

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